

Black Hole Condensation and Duality in String Theory¹

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The classical, four-dimensional theories derived by Calabi-Yau compactification of string theory bear a striking resemblance to the real world [1]. However, these classical theories are beset by several serious difficulties:

1. There are too many of them. This is aesthetically displeasing because a unified theory should be unique. It also entails a loss of predictive power.
2. The theory breaks down and develops naked singularities at certain “conifold” points in the moduli space of the massless four-dimensional scalar fields [2].

In this talk we shall argue, in the context of type II string theories, that these problems are in part resolved by nonperturbative quantum effects. Thus — unlike e.g. nonabelian gauge theories — string theory needs quantum mechanics for consistency. This suggests that the fundamental formulation of quantum string theory may not take the usual form which begins with a classical theory followed by quantization. Rather string theory may be intrinsically quantum in nature and not have a consistent classical limit.

The structure of conifold singularities is an old and beautiful subject in algebraic geometry. The mathematical description will not be repeated here. Relevant aspects and references can be found in [3]. The basic picture is as follows. The space of Calabi-Yau string vacua is the moduli space of Ricci-flat metrics on the Calabi-Yau. For each coordinate Z^i on the moduli space there is a massless 4D scalar $Z^i(x)$ which describes how the size and shape

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of the Calabi-Yau vary in spacetime. These moduli fields are governed by the 4D effective action

$$\mathcal{L}_{\text{eff}} = \int d^4x \sqrt{-g} G_{ij}(Z) \nabla_\mu Z^i \nabla_\nu Z^j g^{\mu\nu} . \quad (1)$$

where g is the metric on spacetime and G is the metric on the moduli space.

The moduli space metric G is classically determined from Calabi-Yau data [4]. In the (type II) context which we consider, there are no quantum corrections to G due to $N = 2$ supersymmetric nonrenormalization theorems.

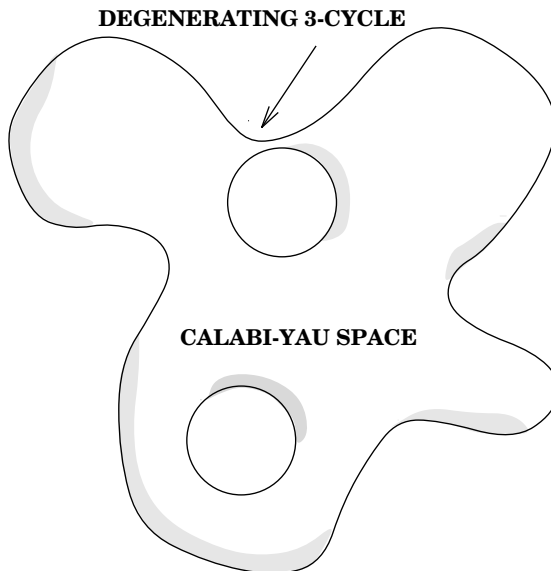


Figure 1: Near a conifold, a minimal 3-cycle degenerates to zero volume and the Calabi-Yau space develops a singular node. (The handles are meant to indicate the complex topology involved: real Calabi-Yau spaces have $\pi_1 = 0$.)

The Z^i 's measure the size of topologically non-trivial, minimal-volume cycles (i.e. submanifolds) embedded in the Calabi-Yau [5]. To be definite, let us consider minimal 3-cycles. At “conifold” points in the moduli space, the minimal volume of a topologically non-trivial 3-cycle can actually shrink to zero. We can choose local coordinates so that the conifold singularity is at $Z^1 = 0$. At $Z^1 = 0$ the Calabi-Yau develops a singular node and is no longer a smooth manifold as depicted in figure 1. Conifold singularities generically occur at finite distances in the moduli space.

It is perhaps not surprising that the moduli space metric G in (1) itself turns out to be logarithmically singular at $Z^1 = 0$ [2]

$$G \sim \ell n|Z^1| . \quad (2)$$

This is a real curvature singularity and cannot be eliminated by a coordinate transformation of the Z^i 's. Thus classical string theory breaks down whenever a moduli field happens to run into a conifold singularity. It can be seen that these singularities are real codimension two so it is hard to avoid such collisions.

A curvature singularity is not the only suspicious behavior of type II string theory near a conifold. These theories have extremal, charged black holes whose mass can be exactly determined using $N = 2$ supersymmetry. These masses are proportional to

$$M_{\text{BH}} = |Z^1| . \quad (3)$$

Hence the black hole becomes massless at the conifold singularity $Z^1 = 0$.³

The black hole degenerates to zero mass for a simple reason. It began life in ten dimensions as a black 3-brane [6]. This is an extended black hole whose horizon is topologically $R^3 \times S^5$, and with a constant mass per unit three volume. In a Calabi-Yau compactification these 3-branes can wrap around a non-trivial 3-cycle. To a low-energy 4D observer, such a configuration will appear to be an ordinary extremal black hole with mass proportional to the volume of the 3-cycle. When this volume degenerates at a conifold the 4D mass will degenerate along with it.

To summarize the picture so far, the conifold is characterized by

$$\begin{aligned} Z^1 &\rightarrow 0 , \\ M_{\text{BH}} &\rightarrow 0 , \\ \mathcal{L}_{\text{eff}} &\rightarrow \infty . \end{aligned} \quad (4)$$

In fact this situation is not as disturbing or unusual as it seems. It is well known that massless particles produce singularities in low-energy effective

³ Far from the conifold the black holes are well-described by semiclassical solutions with horizons. However, in a neighborhood of the conifold, its Compton wavelength exceeds its Schwarzschild radius and the semiclassical description breaks down.

actions due to infrared divergent loop integrations. A non-singular description of the physics can be found in a Wilsonian effective action $\mathcal{L}_{\text{eff}}^w$. This is obtained (in principle) by starting from the exact microscopic theory and integrating out fluctuations of all fields — massive and massless — down to some Wilsonian cutoff M_c , well below the Planck or string scales, as depicted in figure 2. This action differs from the 1PI (one-particle-irreducible) effective action \mathcal{L}_{eff} usually discussed in string theory in which fluctuations of all wavelengths are integrated out. Divergences in the 1PI effective action arise in integrating out fluctuations of massless fields from M_c down to zero energy. Computations of a scattering process with external momenta of order p using $\mathcal{L}_{\text{eff}}^w$ involves a quantum loop expansion with loop momenta cutoff at M_c . Infrared divergences will then typically be controlled by the external momenta p , and the computation will yield a finite answer.

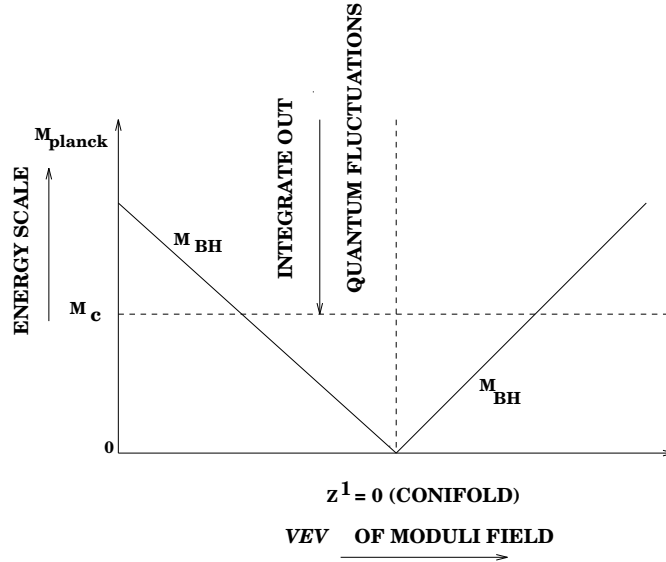


Figure 2: *The smooth Wilsonian effective action is defined by integrating out quantum fluctuations of all fields down to the cutoff M_c . No matter how low M_c is, there is always a region in the moduli space surrounding the conifold in which black holes are lighter than M_c and must be included in the Wilsonian action.*

In the case of conifold singularities, the divergences in \mathcal{L}_{eff} have precisely the right coefficients to have been produced by integrating out a black hole [3]. This has remarkably been confirmed even for subleading terms in \mathcal{L}_{eff} [7]. We

conclude that the underlying Wilsonian effective action has couplings which are nonsingular as $Z^1 \rightarrow 0$. Finite-momentum processes can be computed at the conifold utilizing $\mathcal{L}_{\text{eff}}^w$. Hence we see that classical inconsistencies of string theory are cured by quantum loops of black holes. It is fascinating that the demand for a consistent theory forced us to include these black holes with virtual fluctuations on the same footing as elementary strings.

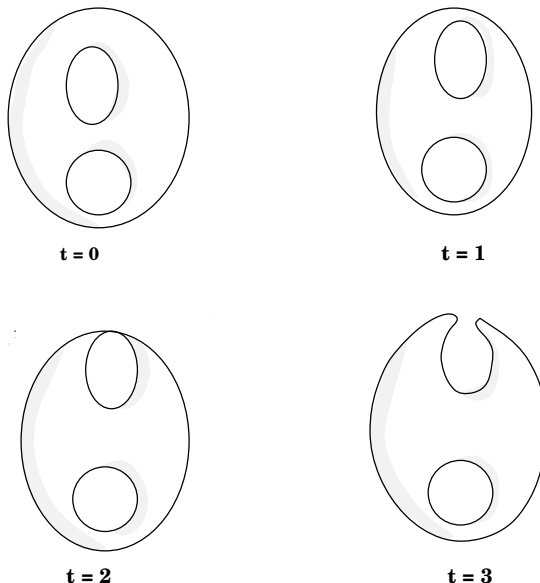


Figure 3: *The shape of a Calabi-Yau space slowly changes and develops a node at time $t = 2$. Black holes then condense, implementing a smooth transition to a topologically distinct Calabi-Yau space at time $t = 3$.*

The appearance of a massless particle often signals a phase transition. One may wonder if there is a new phase of string theory characterized by

$$\langle \Phi_{\text{BH}} \rangle \neq 0 , \quad (5)$$

where Φ_{BH} is the field whose quanta are the degenerating black holes. This may seem like a difficult question, but in fact the answer is easily determined using $N = 2$ supersymmetry, which fixes the potential for the field Φ_{BH} . In the simple conifold singularities described in [3], the answer is no: black hole condensation is prevented by a quartic potential.

The situation is dramatically different for the more complex conifold singularities analyzed in collaboration with Brian Greene and Dave Morrison [8]. These singularities correspond to multiple degenerations at which P 3-cycles degenerate and P black holes come down to zero mass. The Wilsonian action at the singularity involves P black hole fields, Φ_{BH}^A , $A = 1, \dots, P$. $N = 2$ supersymmetry again determines the potential $V(\Phi_{\text{BH}}^A)$. In some cases it is found that V has flat directions along which black holes can condense!

It might appear that a new branch of the string moduli space has been discovered. However, there is overwhelming evidence that $\langle \Phi_{\text{BH}}^A \rangle \neq 0$ branches are not new string vacua. Rather they are a new, dual description of old string vacua. The spectrum of massless particles in the $\langle \Phi_{\text{BH}}^A \rangle \neq 0$ branches agree in each of the thousands of known examples with the spectrum of a known Calabi-Yau space. Furthermore, pairs of Calabi-Yau's which are connected in this manner by black hole condensation are the same as those pairs previously known from the work of [10] to be connected by a singular conifold transition in which an S^3 is shrunk to zero size and then blown back up as an S^2 . Hence black hole condensation in four dimensions corresponds to a change in the topology of the internal Calabi-Yau, as depicted in figure 3. In general relativity the topology of a manifold cannot change in a smooth fashion. String theory is an extension of general relativity in which smooth topology change can occur.

Thousands, and possibly all simply-connected, Calabi-Yau's are connected by such transitions. In this fashion the plethora of disconnected string vacua are unified into a smaller number — possibly one — of moduli spaces as illustrated in figure 4. The long-term aspiration is that, when understood, the dynamics of supersymmetry breaking will select a preferred point(s) in this space.

In the “old”, Calabi-Yau, description of the $\langle \Phi_{\text{BH}}^A \rangle \neq 0$ phase, Φ_{BH}^A is identified as a field whose quanta are fundamental strings rather than black holes. Thus under the topology-changing phase transitions,

$$\begin{aligned} \text{Black Holes} &\rightarrow \text{Strings} \\ \text{Strings} &\rightarrow \text{Black Holes} . \end{aligned}$$

Black holes and strings are dual descriptions of the same entity. For decades theorists have pursued the idea that elementary particles are secretly black holes. We have seen that a version of this idea is realized in string theory.

Hence string theory succeeds not only in unifying all particles and forces with one another, but in unifying them with black holes as well.

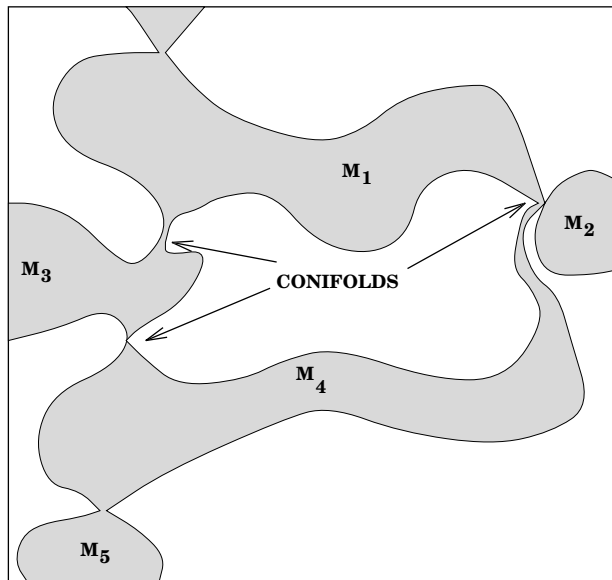


Figure 4: The vacuum moduli spaces, M_1, M_2, \dots of topologically distinct Calabi-Yau spaces are branches of a larger moduli space connected via conifold transitions.

The preceding discussion has close parallels in the beautiful work of Seiberg and Witten on $N = 2$, $d = 4$ gauge theories [11, 12]. In the pure $SU(2)$ gauge theory, [11] there is a conifold singularity which appears at a special point in the moduli space of Higgs *vev*'s. This theory also contains 't Hooft-Polyakov monopoles which degenerate at the conifold. The Wilsonian theory including light monopoles is smooth at the conifold, just like the Wilsonian theory with light black holes described here.

There are also some apparent differences with the work in [11]. The conifold singularity of [11] has an alternate description as a divergence in the *quantum* sum over Yang-Mills instantons, as opposed to the Calabi-Yau conifolds which have an alternate description (utilizing mirror symmetry) as a *classical* sum over worldsheet instantons. This distinction evaporates in the context of a dual description of string theory in which the string is itself a soliton [13]. In such a description the classical worldsheet sum becomes a quantum sum over spacetime instantons [3, 14]. Explicit examples of this

have been understood in the context of dualities relating type II-heterotic string compactifications [15]. Duality promotes the analogy to an identity: the dual transforms of black holes are monopoles, and worldsheet instantons turn into Yang-Mills instantons.

Field theory analogs of the conifold transitions in which the topology of the Calabi-Yau changes also exist [12]. For example, in the $N = 2$ $SU(2)$ gauge theory with two flavors of “quarks” in an $SU(2)$ doublet, the moduli space has several branches. The first is called the Coulomb branch, along which $SU(2)$ is broken to $U(1)$ by an adjoint Higgs vev and all quarks are massive. At special conifold points on the Coulomb branch massless charged states appear. These can condense and form a new branch called the Higgs branch along which the $U(1)$ is broken. Condensation of these massless charged states creates a new branch of the gauge theory moduli space in the same fashion that black hole condensation creates a new branch of the string moduli space.

String dualities again promote the analogy to an identity. From the dual, heterotic perspective, the exotic topology-changing conifold transitions of the type II theory are nothing but condensation of various light-charged fields: The moduli space of $N = 2$ heterotic string vacua contains many special points where charged perturbative string states become light and condense, changing both the massless spectrum and dimension of the moduli space. Hence, a consistent picture of heterotic-type II string duality relies crucially on the existence of black hole condensation in type II theories.

Perhaps the most exciting aspect of recent developments is the deep new puzzles they have raised. I would like to draw attention to one of these puzzles related to the preceding analogy. Consider a moduli field which is slowly rolling in a generic fashion and encounters a conifold singularity. Part of spacetime will spill across the transition, and a bubble of the new phase will form. Inside the bubble a new spectrum of massless particles will appear. Our analysis of the low-energy effective action enables one to obtain the lowest-order approximation to this process.

However, in a complete theory one should, in principle, be able to compute arbitrarily high order corrections to the leading approximation. Clearly, the usual string perturbation rules are useless here because different conformal field theories are relevant to the regions inside and outside the bubble. We do not have a rule for computing these corrections. It is furthermore clear that, whatever those rules are, they are quite different from the usual rules

of string theory.

The analogy with the Seiberg-Witten field theory case is again illuminating. In that case the low-energy effective theories on the Higgs and Coulomb branches can be used to give a leading-order description of the formation of a bubble of the Higgs branch inside the Coulomb branch. However, a systematic computation of the corrections can only be made from knowledge of the microscopic $SU(2)$ gauge theory.

In our current understanding of string theory, it is as if we have seen the last equations in the papers of Seiberg and Witten which describe the low-energy effective abelian gauge theories. To fully understand string theory, we must work backwards from these last equations to the first equations in which the theory is fundamentally defined as an $SU(2)$ gauge theory.

Clearly this is an enormous task. At the same time, recent developments have provided us with new tools and concrete questions with which we can address these issues, and progress is being made in leaps and bounds. It is an exciting time for string theory.

ACKNOWLEDGMENTS

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